Number – W/C 1st June 2020

We start on fairly familiar territory with some key Number Vocabulary, most of which you have met before; but still can get muddled with.

Make relevant notes in your books and write the answers to any questions in your books.

Multiple: in the multiplication table. For example, the multiples of 7 are 7, 14, 21, 28, 35, …

Factor: Any number that goes exactly into a given number. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12. (It is best to do this in pairs starting from 1)

Prime number: Any number that only has two factors, 1 and itself. Note that 1 is not a prime number, as it only has 1 factor. It is very important that you recognise the prime numbers up to about 31 as you will be using them later in product of primes

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 …… (2 is the only even prime number, because 2 is a factor of all other primes!)

LCM (Lowest common multiple): – this is the smallest number that is the multiple of 2 or 3 given numbers.

e.g. the LCM of 5 and 6 is 30
the LCM of 4 and 6 is 12

You can find this by listing the multiples of each number and finding the smallest one. For much bigger numbers we can write each number as a Product of Primes and use Venn Diagrams to find the LCM and HCF (see below)

HCF (Highest Common Factor): - this is the biggest number that is a factor of 2 or 3 given numbers.

e.g. the HCF of 8 and 12 is 4

You can find this by listing the Factors of each number or use products of primes as mentioned above.

Reciprocal: The **reciprocal** of any number is 1 divided by the number.

e.g. The reciprocal of 2 is 1 ÷ 2 = ½ (or 0.5)

The reciprocal of 0.25 is 1 ÷ 0.25 = 4

You can find the reciprocal of a fraction by swapping the numerator and the denominator.

For example: The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

The reciprocal of $\frac{7}{4}$ is $\frac{4}{7}$.

To find the reciprocal of decimals without a calculator it is often easier to turn the decimal into a fraction and flip it over!

e.g. To find the reciprocal of 0.4

0.4 = $\frac{4}{10}$ so the reciprocal of 0.4 is $\frac{10}{4} = \frac{5}{2} = 2.5$

To find the reciprocal of a Mixed Number turn it into an Improper Fraction then flip it over!

e.g. To find the reciprocal of 2.75

2.75 = $2\frac{3}{4} = \frac{11}{4}$ so the reciprocal of 2.75 is $\frac{4}{11}$ (check this on your calculator 1 ÷2.75)

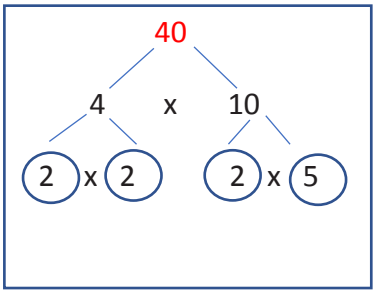
Product of Primes:

Every integer (*whole number*) has a unique prime factorisation, so it doesn't matter which factors are chosen to start the factor tree as you will end up with the same answer.

Example 1: Write 40 as a product of its prime factors.

Firstly, find two numbers that will multiply together to give 40. For example, 4 x 10 = 40 would be one way of starting this calculation.

Neither 4 nor 10 is a prime number, and this question is looking for prime factors, so each number must be broken down again into factor pairs. Continue breaking down the factors into factor pairs until you are only left with prime numbers. Then circle these prime numbers.

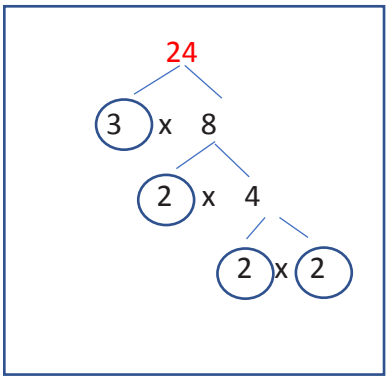


The question has asked for a **product of prime factors**. Write all of the circled prime numbers (found in the prime factor tree) as a product.

So, 40 = 2 x 2 x 2 x 5 or using indices we can write $40 = 2^3 \times 5$

Remember to put the x between each prime number (product means times)

Example 2: Write 24 as a product of its prime factors



As before, circle the prime numbers and find a pair of numbers that multiply together to give you the numbers that are still not prime.

So, 24 = 2 x 2 x 2 x 3 *(check this is correct)*

Using Indices, we can write:
$24 = 2^3 \times 3$

Finding HCF and LCM

Above we mentioned finding the Highest Common Factor (HCF) and the Lowest Common Multiple (LCM) by simply listing them for the numbers involved and finding the Highest (for Factors) or Lowest (for Multiples) in each list.

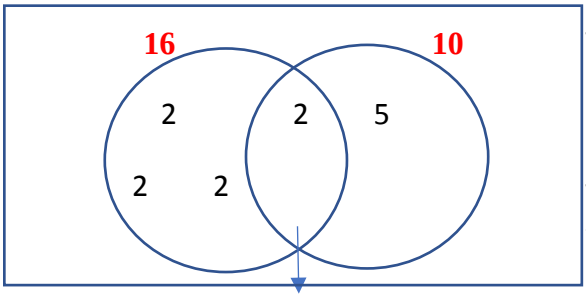
A more efficient way, especially for bigger numbers is to use Product of Primes and show the information in a Venn Diagram.

Example 3: Find the HCF of 16 and 10

- First write both numbers as a product of primes (as above)

  16 = 2 x 2 x 2 x 2
  10 = 2 x 5

- Then put this information in a Venn Diagram



The HCF is the product of the numbers in the intersection *(The numbers in the centre bit multiplied together)*

The LCM is all of the numbers multiplied together

HCF = 2

LCM = 2 x 2 x 2 x 2 x 5 = 80

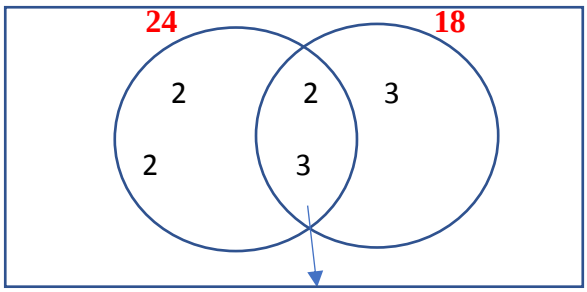
(NOTE this is not the same as 16 x 10 because the 2 is in both so is only counted once!)

Example 3: Find the HCF of 24 and 18

- First write both numbers as a product of primes (as above)

  24 = 2 x 2 x 2 x 3
  18 = 2 x 3 x 3

- Then put this information in a Venn Diagram



HCF = 2 x 3 = 6

The HCF is the product of the numbers in the intersection *(The numbers in the centre bit multiplied together)*

The LCM is all of the numbers multiplied together

LCM = 2 x 2 x 2 x 3 x 3 = 72

Read this through again and make notes or if you can print this off and stick it into your book. Then work through the 3 tasks below:

Task 1: Watch the following clips on mathswatch:

Clip 76	Reciprocals
Clip 78	Product of Primes
Clip 79	HCF
Clip 80	LCM

Task 2: Work through the following questions:

Write the answers in your book, showing any relevant workings i.e. the list of factors or the first few multiples until you find the same number in both lists *(common to both)*

Question 1.

Hot-dog sausages are sold in packs of 10 and hot-dog buns are sold in packs of 8. How many of each do you have to buy to have complete hot dogs with no wasted sausages or buns?

Hint: Find the LCM of 8 and 10

Question 2.

A bell chimes every 6 seconds. Another bell chimes every 5 seconds. If they both chime together, how many seconds will it be before they both chime together again?

What hint would you give someone?

Question 3.

Fred runs round a running track in 4 minutes. Debbie runs round in 3 minutes. If they both start together on the line at the end of the finishing straight, when will they both be on the same line together again? How many laps will Debbie have run? How many laps will Fred have run?

Question 4.

Write the following as a Product of Prime Numbers, giving your answers in index form: *Draw the trees in your book and show the answer clearly – the tree is the working not the answer*

1. 24 =
2. 40 =
3. 35 =
4. 18 =
5. 28 =
6. 42 =
7. 60 =
8. 124 =
9. 300 =
10. 54 =

Question 5:

Use a Venn Diagram to find the HCF and LCM of the following:

1. 32 and 12
2. 16 and 20
3. 35 and 18
4. 24 and 30
5. 30 and 42
6. 70 and 56

Task 3: Finally, please complete the assignment that I have set you on mathswatch