Standard Form – W/C 8th June 2020

We touched on Standard Form very briefly before your last test but it was evident that most of you had not really understood how to write numbers in Standard Form and were unable to perform calculations with Standard Form. So, the challenge this week is to make sure you feel completely comfortable with every aspect of it. I have written lots of notes to help you. Please read them carefully!

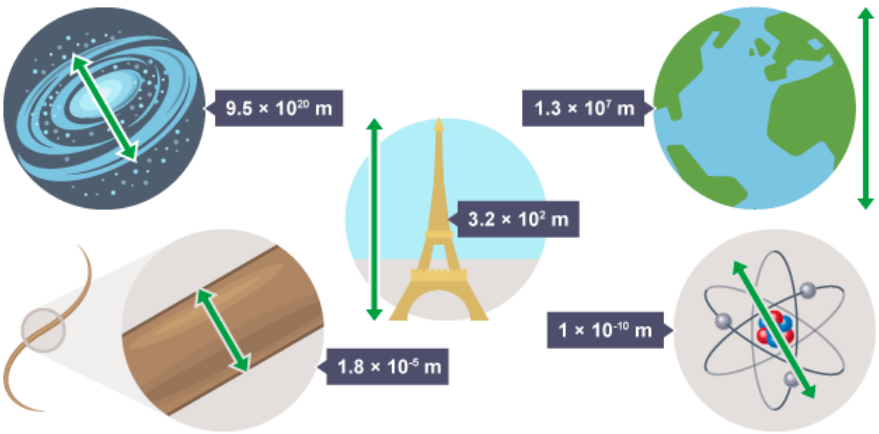
Standard form, is a system of writing numbers which can be particularly useful for working with very large or very small numbers. It is based on using powers of 10 to express how big or small a number is.

Standard form is written in the form of:

$a \times 10^n$ where $1 \leq a < 10$ (a is bigger or equal to 1 but less than 10)

and n is an integer (any positive or negative whole number)

For example, 3.1×10^{12}



Scientists use standard form when working with the speed of light and distances between galaxies, which can be enormous. The size of bacteria or atoms may also be referred to in standard form as they are so tiny.

Converting between ordinary numbers and standard form

To convert a number into standard form, split the number into two parts - a number multiplied by a power of 10.

Powers of 10

Standard form uses the fact that the decimal place value system is based on powers of 10:

$10^0 = 1$	$10^3 = 1000$	$10^6 = 1000\,000$
$10^1 = 10$	$10^4 = 10\,000$	$10^7 = 10\,000\,000$
$10^2 = 100$	$10^5 = 100\,000$	$10^8 = 100\,000\,000$

Large numbers

Example Write 50,000 in standard form.

50 000 = 5 x 10 000 = 5 x 10^4

Note: 34 x 10^7 is not written in standard form – why?

Hopefully, you have spotted that the first number is not between 1 and 10.

To correct this, we need to divide 34 by 10. To balance out the division of 10, we also need to multiply the second part by 10, which gives 10^8.

34 x 10^7 and 3.4 x 10^8 are equal in value, but only the second is written in standard form.

Example What is 87 000 in standard form?

87 000 = 8.7 x 10 000 = 8.7 x 10^4 (I just count the number of places I have moved the decimal point by putting my finger between the 8 and 7 and jumping to the end!)

8 7 0 0 0 4 jumps – so the answer is 8.7 x 10^4

1 2 3 4 Note: we had to start with a number between 1 and 10, so could not use 87 or 0.87

Small numbers

Look at the patterns to try to understand negative indices (something we will be doing later!)

$10^0 = 1$

$10^{-1} = 0.1$

$10^{-2} = 0.01$

$10^{-3} = 0.001$

$10^{-4} = 0.0001$

$10^{-5} = 0.00001$

$10^{-6} = 0.000001$

A negative power does not mean that the number is negative. It means that we have gone from multiplying by 10 to dividing by 10.

Example Write 0.0005 in standard form.

0.0005 can be written as 5 x 0.0001 = 5 x 10^{-4}.

As before I count the jumps – this time backwards! 0 . 0 0 0 5

Notice I start from the decimal point that would be after the units – 4 jumps back so 5 x 10^{-4}

More Examples:

0.03 = 3 x 10^{-2}

0.000039 = 3.9 x 10^{-5}

Converting from standard form

To convert a number in standard form to an ordinary number, simply do the multiplication.

Examples

1.34 x 10^3 = 1.34 x 10 x 10 x 10 = 1 340

(just move decimal point 3 places to the right – number gets bigger)

4.78 x 10^{-3} = 4.78 x 0.001 = 0.00478

(just move decimal point 3 places to the left – number gets smaller)

2.99 x 10^7 = 29 900 000 (move decimal point 7 places to the right)

1.36 x 10^{-5} = 0.0000136 (move the decimal point 5 places to the left)

Ordering numbers in standard form

Numbers written in standard form can be ordered by first looking at the power of 10, which tells you the size of the numbers. If two or more numbers have the same power of 10, use the first part of the number to decide the order.

Example

Write these numbers in ascending order:

7×10^{-2} , 3.2×10^2 , 4.1×10^4 , 3.81×10^{-5} , 5.6×10^3 , 2×10^4

As the numbers 4.1×10^4 and 2×10^4 have the same power of 10 we look at the number in front and can see that 2×10^4 is the smallest. For all of the others we can just look at the powers and put them in ascending order (remember that -5 is less than -2) *see below*

So the answer is 3.81×10^{-5} , 7×10^{-2} , 3.2×10^2 , 5.6×10^3 , 2×10^4 , 4.1×10^4

Compare the powers	-5	-2	2	3	4	4

Calculating standard form without a calculator

When **adding and subtracting** standard form numbers, you have to:

1. convert the numbers from standard form into ordinary numbers
2. complete the calculation
3. convert the number back into standard form

If the power of 10 is the same for both you can just add or subtract the numbers, if it is not you can always change it to be the same. However, to avoid mistakes it may be easier to follow the steps above – which is Method 1

METHOD 1 (As per the steps above)	METHOD 2
Calculate $(4.5 \times 10^4) + (6.45 \times 10^6)$	Calculate $(4.5 \times 10^4) + (6.45 \times 10^6)$
$= 45\,000 + 6\,450\,000$ *line up in columns*	*Divide by 100* *Multiply by 100*
$= 6\,495\,000$	$= (0.045 \times 10^6) + (6.45 \times 10^6)$
$= 6.495 \times 10^6$ *put the answer back into standard form*	$= 6.495 \times 10^6$

When multiplying and dividing you can use index laws:

1. multiply or divide the first part of the numbers
2. apply the index laws to the powers of 10 *(we have done this but more to come soon)*
3. check whether the first part of the number is between 1 and 10

To multiply powers of 10, add the powers together, eg $10^6 \times 10^4 = 10^{6+4} = 10^{10}$. To divide powers of 10, subtract the powers, eg $10^7 \div 10^2 = 10^{7-2} = 10^5$.

Example 1.

Work out $(3 \times 10^3) \times (2 \times 10^9)$ When you multiply the order does not matter e.g. 3x8 = 8x3

So, this is the same as $3 \times 2 \times 10^3 \times 10^9 = 6 \times 10^{12}$

$3 \times 2 = 6$ $10^3 \times 10^9 = 10^{3+9} = 10^{12}$

As 6 is between 1 and 10, this number is in standard form.

Example 2.

Work out $(4 \times 10^8) \times (7 \times 10^{-3}) = 4 \times 7 \times 10^8 \times 10^{-3} = 28 \times 10^{8+-3} = 28 \times 10^5$

BUT 28 is not between 1 and 10, so 28×10^5 is NOT in Standard Form. To convert this to standard form, divide 28 by 10 so that it is a number between 1 and 10. To balance out this out, multiply the second part by 10 which gives 10^6.

So the final answer is $(4 \times 10^8) \times (7 \times 10^{-3}) = 2.8 \times 10^6$

The rules are similar for dividing but I encourage you to re-write the sum in the form of a fraction and then simplify! May sound harder but it seems to make more sense when you do it!

Example 3.

Work out $6.4 \times 10^6 \div 3.2 \times 10^3 = \frac{6.4 \times 10^6}{3.2 \times 10^3} = 2 \times 10^{6-3} = 2 \times 10^3$ *Just watch out for the double negative if you are dividing by a negative index.*

Example 4.

Work out $4 \times 10^4 \div 8 \times 10^{-2} = \frac{4 \times 10^4}{8 \times 10^{-2}} = 0.5 \times 10^{4--2} = 0.5 \times 10^6$ *Can you spot what is wrong with this answer?*

As 0.5 does not lie between 1 and 10 it is not in Standard form, so we have to multiply the 0.5 by 10 and to keep the number the same make the 10^6 , 10 times smaller i.e. 10^5

So, the final answer is $4 \times 10^4 \div 8 \times 10^{-2} = 5 \times 10^5$

Finally: Calculating standard form with a calculator

Standard form can be calculated on a scientific calculator with the 'exponent' button. The exponent button may be shown as EXP, EE or $x10^x$. For example, to input 1.2×10^7 press 1.2 EXP 7 or 1.2 {$x10^x$} 7 Have a go on your own calculator – the button is usually at the bottom next to the ANS and = buttons.

Task 1: Just in case you have not done this ….Read all of the instructions above and work through the examples.

Task 2: Watch the mathswatch clip 83

Task 3: Complete a selection of questions from your on-line book from Pages 32 Exercise 1 and Exercise 33 Exercise 2 and Exercise 3. Many of you have done the first two exercises so you may wish to focus on doing as many as you can from Exercise 3

Task 4: Complete the Mathswatch Assignment

Hopefully this topic will be helpful in Science too!