Trigonometry – W/C 15th June 2020

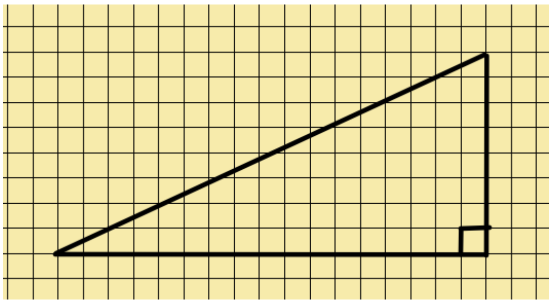
This will be a brand-new topic for all of you and one that you may find very confusing to start with. Once you become familiar with the new notation, it will start to feel better. As we will be meeting for an hour a week from this week, I will be able to help you more easily.

So, we begin with an Introduction to Trigonometry.

Trigonometry is derived from the Greek words for 'triangle' (*trigonon*) and 'measure' (*metron*). We have already explored triangles through construction and Pythagoras' Theorem, there is even more to learn in the magical world of triangles...so here we go!

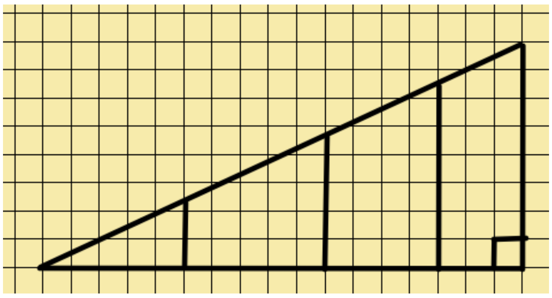
We are going to start with an investigation. I recommend that you read through the whole of Task 1 first and then begin. Don't worry if the investigation confuses you, as there is a method that we will learn. This is just supposed to be a bit of 'discovery' and 'fun'! You will need a ruler and a protractor.

TASK 1: Draw your self a large RIGHT ANGLED TRIANGLE in your book, like this:



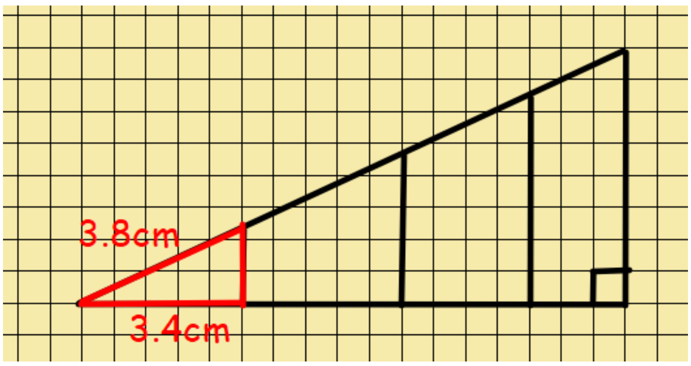
You will remember from when you did Pythagoras' Theorem that the longest length (the one opposite the Right Angle) is called the Hypotenuse.

Now divide up your triangle into smaller triangles so that you have a selection of similar triangles.



We are going to investigate a relationship between the lengths of the various similar triangles that you have drawn.

I have measured the lengths of my first little triangle. Be as acurate as possible when measuring your triangles.



In this investigation you are looking at the ratio of the side lengths. My (smaller ÷ larger) ratio is:

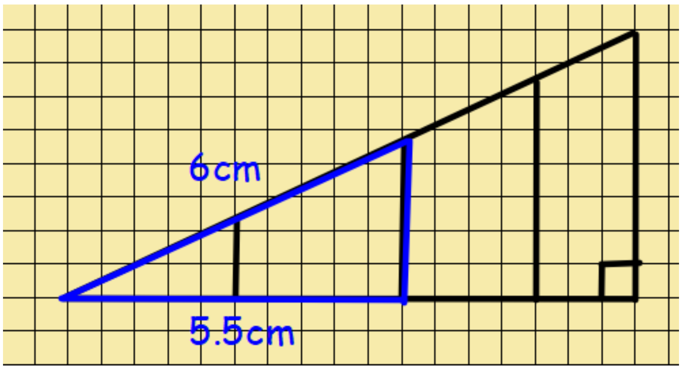
$$\frac{3.4}{3.8} = 0.89$$

(your lengths will obviously be different to mine)

Make a note of your calculation in your books.

Diagrams are not *to scale**

Next triangle:



When I find the same ratio, dividing the smaller number by the larger number, I get this:

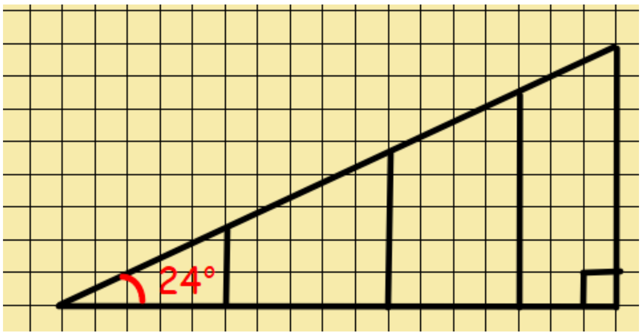
$\frac{5.5}{6}$ = 0.92

When I continue to measure my other side lengths and divide the smaller measurement by the hypotenuse I get:

$\frac{8.3}{9}$ = 0.92 and $\frac{9.7}{10.5}$ = 0.92

I am starting to notice a pattern (and I am also questioning whether my measuring was entirely accurate on my first triangle!)
Are you getting answers that are very close to each other? – if not you may want to check the accuracy of your measuring!

Now for the clever bit!
Measure your angle at the bottom corner.



Mine measures 24 degrees – yours will be different.

So now type into your calculator cos24
Make sure your calculator is in degrees mode*
Cos24 = 0.91
Your calculator knows what the ratio will be when given the angle! (Allowing for a little accuracy error)

***Information on how to get your calculator in degrees mode is explained in the Mathswatch video**

What this means is that for every right-angled triangle that has an angle of 24°, from the tiniest to a big one drawn on the school playing field, the relationship between the length of the bottom line and the hypotenuse is fixed and will always be 0.91.

As you will have a different angle, you will have a different ratio but it should be the same (or very close) for all four triangles.

OK, that sounds great but what is the point of all this? This knowledge allows us to find the length of missing sides and angles in Right Angled Triangles.

So, for my Triangle that has an angle of 24° I know that $\frac{bottom\ line}{hypotenuse}$ = cos24 = 0.91

I can use this to find the Length of the Hypotenuse if the bottom line was 12cm.

After rearranging my formula:

$$\text{Hypotenuse} = \frac{bottom\ line}{\cos 24}$$
$$= \frac{12}{\cos 24}$$
$$= 13.1\text{cm (1dp)}$$

So, if I extend my triangles, I could check this is the case!

This could be quite handy in architecture and construction. Trigonometry is used in mechanics, biology, engineering, navigation, computer graphics, you name it!

Feel free to experiment some more, by drawing more triangles.

This is just the start. We have used the cosine of an angle (cos) but you will see from your calculator there is some other buttons sin (sine) and tan (tangent). These both refer to a different ratio of the sides of the triangle.

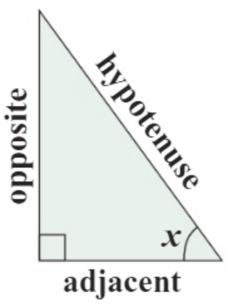
Do not panic if this seems like 'double dutch' at the moment – all will become clear. This was just supposed to be a bit of fun!

If you have not had the equipment to do the above or have chosen not to, it is not a problem – just start from here:

TASK 2: Before we begin using Trig (friendly and easier way of writing Trigonomentry!) to find missing lengths and angles, you need to confident labelling our triangle. Watch this video:
https://corbettmaths.com/2013/03/30/trigonometry-introduction/

Labelling your triangle correctly is of huge importance when doing trigonometry questions.

Copy this into your books:



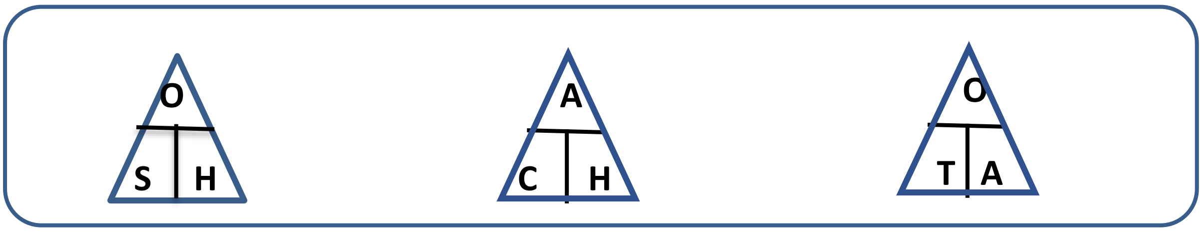
In a right-angled triangle, the side opposite the right angle is called the **hypotenuse**, the side opposite an angle is called the **opposite** and the side next to an angle is called the **adjacent**.

The three sides are linked by the following formulas:

$$\sin x = \frac{\text{opp}}{\text{hyp}}, \quad \cos x = \frac{\text{adj}}{\text{hyp}}, \quad \tan x = \frac{\text{opp}}{\text{adj}}$$

Just have a quick practice at labelling some triangles. Go onto Mathswatch and watch clip 168. You only need to watch the first 2 minutes and practice labelling the sides of triangles according to where the angle is positioned.

If you like using Formula Triangles to rearrange formula then you can write them as:



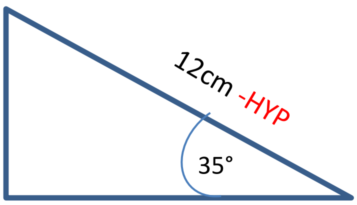
Learn - SOH CAH TOA   Learn to write it as $S^{O}H$ $C^{A}H$ $T^{O}A$

Trigonometry-Finding missing sides

Continue watching the Mathswatch clip 168. You can watch from the beginning again as a recap, or skip to 2 minutes in. Watch carefully and draw out the first example, including the 4-step method shown in the video. Don't rush this example, feel free to rewind as many times as you need. Then have a go at the next 3 examples in the video, drawing each one out in your books neatly and showing each step of the working. Create helpful and clear notes for yourself as it will help you in the future. Stop the video at 7 minutes. We are saving Angles for next week!

We are going to continue practicing finding missing sides for this lesson.

Now watch this video if you're still feeling a bit uncertain:
https://corbettmaths.com/2013/03/30/trigonometry-missing-sides/



Model Example

Find x to 1 decimal place

X - ADJ

1. First label the sides (In this case we know the **HYP** and need to find the **ADJ** but don't care about the OPP!
2. Decide which ratio to use. As we do not need the OPP the ratio is Cos (Angle) = $\frac{ADJ}{HYP}$
3. If you are using the Formula Triangles, which I recommend, then cover up the ADJ in cos triangle, this gives X (ADJ) = cos (35) x 12
   X = 9.8298245   **HYP**

You are normally told what to round the answer to, in this case 1 d.p. ALWAYS include the units

So, our answer is X = 9.8cm (1 d.p.)

TASK 3: Complete a selection of questions from the CGP on-line book Pages 222 & 223 Exercise 1, Questions 1, 2 & 3 (In Q2 and Q3 – split the Isosceles triangle into 2 right angled triangles)

TASK4: Complete the assignment on mathswatch.