Sequences – W/C 29th June 2020

Number sequences are sets of numbers that follow **a pattern or a rule**.

First a little bit of vocabulary

If the rule is to add or subtract a number each time, it is called an **arithmetic** sequence.

If the rule is to multiply or divide by a number each time, it is called a **geometric** sequence.

Each number in a sequence is called a **term**.

A sequence which increases or decreases by the same amount each time is called a **linear sequence**.

Term to term rules

The **term to term rule** of a sequence describes how to get from one term to the next.

Example 1

Write down the term to term rule and then work out the next two terms in the following sequence.

3, 7, 11, 15, ...

Firstly, work out the difference in the terms.

This sequence is going up by four each time, so add 4 on to the last term to find the next term in the sequence.

3, 7, 11, 15, **19**, **23**, ...

To work out the term to term rule, give the starting number of the sequence and then describe the pattern of the numbers.

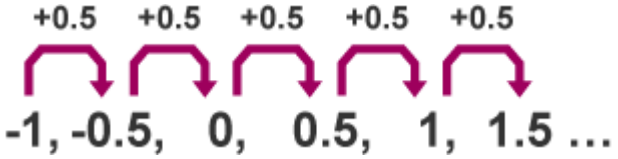
The first number is 3. The term to term rule is 'add 4'. *(Without the starting number we can't generate the sequence)*

Example 2

Write down the term to term rule and then work out the next two terms in the following sequence.

-1, -0.5, 0, 0.5, ...

The first term is -1. The term to term rule is 'add 0.5'.



(Remember sometimes the term to term rule may be subtract a value or multiply by …. or divide by….)

Position to term rules – we call this the n^{th} term where 'n' represents the position in the sequence.

Each term in a sequence has a position. The first term is in position 1, the second term is in position 2 and so on.

Example 1.

Work out the position to term rule for the following sequence: 5, 6, 7, 8, ...

First, write out the sequence and the positions of each term.

Position (n)	1	2	3	4
Term	5	6	7	8

The top row indicates where abouts in the sequence each term is. 5 is the first term or position 1 so n=1

The second row are the values from our sequence. We then have to try and spot a relationship between them.

In this example, to get from the position to the term, we always add 4. We can write this as the n^{th} term is **n + 4**

In other words, we always have to add 4 to the position value to find our term.

e.g the **6**th term would be **6 + 4** = 10 If you continue the sequence above you will see that this is the case

5, 6, 7, 8, 9, 10 … 10 is the 6^{th} term

Example 2 Work out the n^{th} term of the following sequence: 3, 5, 7, 9, ...

Firstly, write out the sequence and the positions of the terms.

Position (n)	1	2	3	4
Term	3	5	7	9

+2 +2 +2

As the rule for going from the position to the term is not obvious, look for the differences between the terms. In this case, there is a difference of 2 each time.

This difference describes **the times tables that the sequence is working in**. In this sequence it is the 2 times tables.

Write out the 2 times tables and compare each term in the sequence to the 2 times tables.

Position (n)	1	2	3	4
2 times table	2	4	6	8
Terms	3	5	7	9

X2

+ 1

To get from the position to the term, first multiply the position(n) by **2** then **add 1.**

We write this as n^{th} term is **2n + 1** **(Remember 2n means 2 x n or n x 2)**

Using the nth term

If the nth term of a sequence is known, it is possible to work out any number in that sequence.

Example 1. Write the first four terms of the sequence with n^{th} term **3n + 4.**

n represents the position in the sequence. The first term in the sequence is when n = 1 , the second term in the sequence is when n = 2 , and so on.
To find the terms, **substitute** the position number for n.

- when n = 1 , 3n + 4 = (3 x 1) + 4 = 7
- when n = 2 , 3n + 4 = (3 x 2) + 4 = 6 + 4 = 10
- when n = 3 , 3n + 4 = (3 x 3) + 4 = 9 + 4 = 13
- when n = 4, 3n + 4 = (3 x 4) + 4 = 12 + 4 = 16

So, the sequence with n^{th} term 3n + 4 is 7, 10, 13, 16 .....

I can see that this goes up in 3's .....I knew it would. How?

Hopefully you knew because it is **3n** (always goes up by 3)

5n – 2 would go up in 5's. The sequence is actually 3, 8, 13, 18, ... (always 2 less than the 5's times table!)

If you put this all together, it will enable us to always find any term for any linear sequence. Example 2: Find the 50th term of 16, 13, 10, 7..

Start with the sequence → **Find an expression for nth term**

16, 13, 10, 7,……

↓

First find the difference between terms — **Our sequence is going down by 3 each time so -3 is rule**

↓

Compare the times table with the sequence — **our times table is (-3x1), (-3x2), (-3x3), (-3x4)**

or -3, -6, -9, -12 …. We compare to our sequence

To go from -3 to 16 we have to + 19

So, **n^{th}** term is **-3n + 19**

To find the 50th term *(rather than writing out 50 terms)* **we can substitute 50 into the expression -3n +19**

ANSWER: 50^{th} term = -3 x 50 + 19 = **-131**

TASK 1: Read the notes above again (or twice if you skipped straight to here!!). Make sure they make sense. It can get a bit confusing between the word position (the place in the sequence) and term (the actual value)

Work through the mymaths lessons: USE GCSE(9-1) England, Algebra, then work through each section of:

G1 - Sequences

G23 - Arithmetic Sequences

G23 - Generating sequences

TASK 2: Watch the following mathswatch clips:

Generating a Sequence Term to Term	**Clip 37**
Generating a sequence form the n^{th} term	**Clip 102 – UP TO 1 minute 40 seconds ONLY** (unless you want to have a go at substituting into harder expressions.
Finding the n^{th} term	**Clip 103**
Fibonacci Sequences	**Clip 141 – UP TO 1 minute 25 seconds ONLY**

TASK 3: Work through the questions below. Write the answers in your book, unless you can print this out and stick it in your book.

1.

Find the next two terms of the following sequences:

a) 3, 7, 11, 15, ☐ , ☐ ,

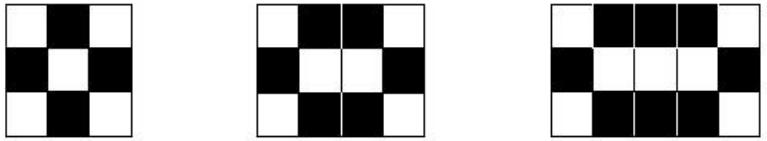
b) 9, 16, 23, 30, ☐ , ☐ ,

c) 25, 21, 17, 13, ☐ , ☐ ,

d) 2, 6, 12, 24, ☐ , ☐ ,

2. A sequence of patterns uses black squares and white squares.

Here are the first three patterns.



Pattern 1 Pattern 2 Pattern 3

(a) Circle the expression for the number of black squares in Pattern *n*.

$4n$ $n + 2$ $6n - 2$ $2n + 2$

(b) Will the number of black squares always be even?

Yes ☐ No ☐

3. A sequence starts

4 8 12 16 ___

The *n*th term is 4*n*

Circle the *n*th term of this sequence

6 10 14 18 ___

4*n* 6*n* 4*n* + 2 6*n* + 2

4. The *n*th term of a sequence is $2n + 1$

The *n*th term of a different sequence is $3n - 1$

Work out the **three** numbers that are

in both sequences

and

between 20 and 40

(Hint: Write out all of the terms in both sequences)

5. Here are the fourth and fifth terms of a Fibonacci-type sequence.

.... 28 43

Each term is the sum of the previous two terms.

6. Which sequence is a geometric progression? *(look at the definition at the start)*

Circle your answer.

1 2 3 4 1 2 4 7

1 2 4 8 1 2 3 5

TASK 4: Finally, complete the mathswatch assignment